Digital Schoolhouse Puzzle Page

"If I had an hour to solve a problem I'd spend 55 minutes thinking about the problem and 5 minutes thinking about solutions"

Albert Einstein

Welcome to the Digital Schoolhouse Puzzle Page

On this paper, we will investigate a series of puzzles that can be used to promote Computational Thinking. This month we will investigate Digital Sums!

Digital Sum

Without the help of a computer or calculator, find the total sum of the digits in all integers from 1 to a million, inclusive.

Answer: The total digit sum in integers from 1 to 10^n, inclusive, is $45 \cdot n \cdot 10^{n-1} + 1$. In particular, for $n = 6$, it is equal to **27,000,001**

It is convenient to disregard 10^n, which contributes 1 to the sum in question, and compute the digit sum in all integers from 1 to 10^{n-1}; we will denote the sum by $S(n)$. It is also convenient to pad every integer smaller than 10^{n-1} by an appropriate number of leading zeros so that all the digits are composed of n digits.

A simple way to find $S(n)$ is to pair numbers together 0 with $10^n - 1$, 1 with $10^n - 2$, and so on. The sum of the digits of each pair is $9n$.

Since there are obviously $10n/2$ pairs, the sum $S(n)$ is equal to $9n \cdot 10^n/2 = 45n \cdot 10^{n-1}$

Thus, for $n = 6$, $S(6) = 45 \cdot 6 \cdot 10^{6-1} = 27{,}000{,}000$ and the total sum of digits is equal to 27,000,001.

Linkage to Computer Science

Pairing the numbers with the same digit sum can be considered either representation change or invariant.

Solutions

Puzzle 55: Easy

Puzzle 56: Medium

Puzzle 57: Hard